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PARAMETERS OF AN ELASTOPLASTIC DILATATION MODEL FOR EARTH MATERIALS

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Strength and dilatation coefficients were taken as a function of the hardening (softening) parameter from data from triaxial tests of specimens of earth materials in compression. It was found that there is a significant change on cohesion, internal friction, and dilatation rate with a change in pressure and initial porosity. The results obtained make it possible to perform numerical calculations of geodynamic processs with multiple internal fractures.

1. The closed mathematical model of the elastoplastic deformation of earth materials includes the momentum balance equation

$$\rho dv_i/dt = \partial S_{ij}/\partial x_j - \partial p/\partial x_i + F_i,$$

where ρ is density; v_i, mass velocity; S_{ij}, stress tensor-deviator; p, pressure; F_i, the body forces and the governing relations characteristic of earth materials. An important element of the latter is allowing simultaneously for internal friction and dilatation.

The laws of flow with hardening used below [1] will be represented in the form

$$\tilde{d}S_{ij}/dt = 2G(\varepsilon_{ij} - \varepsilon\delta_{ij}/3 - \xi S_{ij}),\ \tilde{d}p/dt = -K(\varepsilon - 2\xi\Lambda\tau),$$

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TABLE 1

Classification number	m_0, %	τ_f^0, MPa	$K_1\cdot10^{-2}$	$G_1\cdot10^{-2}$	$\gamma_1\cdot10^3$	s_1	s_2	s_3
Sandstones								
P-0	0,36	120	4,1	2,1	23	0,56	0,66	—
D-9	1	143	2,0	2,0	31	0,56	0,62	—
P-4	1,3	146	3,2	1,5	—	—	0,69	—
P-03	1,5	143	2,0	1,7	19	0,61	0,67	—
P-01	1,7	126	2,5	1,7	—	0,62	0,66	—
P'02	2	159	2,0	1,6	12	0,65	0,70	—
P-5	4	202	1,6	1,6	—	—	0,58	—
P-026	5,3	74	3,1	1,5	—	0,67	0,69	—
P-021	5,7	46	—	—	—	—	0,69	—
VO	6	65	1,2	1,3	4	0,71	0,75	—
NV O	6	71	0,7	0,8	10	0,68	0,71	0,85
Nugget	6	150	—	—	—	—	0,80	—
D-8	7,4	66	1,7	1,3	11	0,60	0,69	—
P-04	18,6	39	2,8	2,4	4	0,71	0,75	—
D-12	—	79	1,6	1,2	—	—	0,74	—
Granites								
Biotite	0,6	90	3,1	2,6	2	0,74	0,77	0,87
Climax granodiorite	0,7	150	—	—	—	0,58	0,75	—
Westerly	0,8	110	—	—	80	—	—	—
Granodiorite Hoggar	—	105	—	—	—	—	0,80	—
Biotitic plagiogranite	1,6	145	2,5	1,6	—	—	—	—
Plagiogranite	2,2	167	1,9	1,5	—	—	—	—
Siltstones								
From hole 328	1,2	91	2,3	1,9	30	0,43	0,48	—
P-4	1,3	137	—	—	—	—	0,69	—
From hole 322	1,67	79	1,7	1,5	—	0,41	0,48	—
» 330	2	76	—	—	—	0,38	0,45	—
P-3	2,1	122	1,7	1,4	—	—	0,70	—
P-2	2,2	168	2,6	1,6	—	—	0,64	—
P-6	2,6	53	5,9	4,4	—	—	0,64	—
P-1	3,6	104	3,6	2,1	—	—	0,70	—
D-19	—	90	2,2	1,5	—	0,65	0,70	—
Marbles								
Marble II	0,11	38	12,4	6,4	0	0,54	0,59	—
Marble I	0,92	60	5,6	2,6	0	0,64	0,69	0,84
Limestones								
D-6	1	92	5,1	2,9	0	0,61	0,65	—
Estonian shale	—	40	3,0	2,2	0	0,50	0,55	—
KMA	17,4	33	7,2	4,5	—	—	0,60	—
Diabases								
№ 5	0,58	84	4,9	3,1	18	0,63	0,65	—
Bratsk hydroelectric power plant	0,98	150	3,7	2,3	19	0,61	0,64	—
Diorites								
№ 11	0,15	208	2,1	1,5	14	0,52	0,56	—
Quartz	0,36	125	—	—	3	0,72	0,76	—

where $\mathcal{D}/dt$ is Jaumann derivative; $\varepsilon_{ij}=(1/2)(\partial v_i/\partial x_j+\partial v_j/\partial x_i)$ is the strain rate tensor; $\varepsilon=\partial v_i/\partial x_i$:

$$\frac{d\lambda}{dt}=\xi=\frac{1}{2}\left(\frac{3G}{4}\frac{S_{ij}\varepsilon_{ij}}{\tau^2}\frac{\partial\varphi}{\partial\tau}-K\frac{\varepsilon}{\tau}\frac{\partial\varphi}{\partial p}\right)\Big/\Delta$$

$$\Delta=G\frac{\partial\varphi}{\partial\tau}-\Lambda K\frac{\partial\varphi}{\partial p}-\frac{1}{2\tau}\frac{d\varkappa_i}{d\lambda}\frac{\partial\varphi}{\partial\varkappa_i}.$$

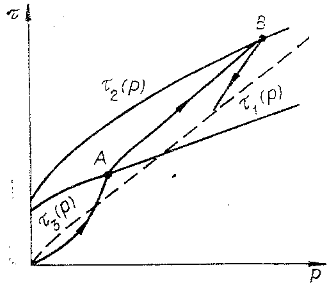

Fig. 1

Here κ_i is the hardening (softening) parameter; τ is the shear stress intensity; the stress

$$\frac{4}{3}\tau^2 = \frac{1}{2} S_{ij}S_{ij},\ S_{ij} = \sigma_{ij} + p\delta_{ij},\ p = -\frac{1}{3}\sigma_{ii}.$$

For the solution to be unique, we need to satisfy the condition $\Delta > 0$. At $\Delta \leq 0$, the continuous development of plastic flow is impossible.

We will seek the condition of plastic loading in the form

$$\varphi(\tau, p, \varkappa_i) = \tau - \tau_i(p) - f_i(p)\varkappa_i = 0,$$

where the subscript i = 1 denotes the process of plastic strain hardening, i = 2 denotes softening, and i = 3 denotes residual strength: $\tau_i(p)$ and $f_i(p)$ represent the corresponding pressure functions. Plastic flow occurs if $\varphi = 0$, $d\varphi = 0$, and $\xi \geq 0$. If $\varphi < 0$ or $\xi < 0$, elastic unloading takes place.

The above-used bulk and shear moduli K and G depend on p and κ_i. The modulus K may also depend on the sign of dp, which makes it possible to consider irreversible volume strain during cubic compression. The dilatation rate $\Lambda = de^p/|d\gamma^p|$ is determined as the plastic increment of the volume due to a unit increment of plastic shear:

$$d\gamma^p = \frac{1}{2} dE^p_{ij}dE^p_{ij},\ de^p = de^p_{ii},\ dE^p_{ij} = de^p_{ij} - \frac{1}{3} de^p_{ij}\delta_{ij}.$$

2. In accordance with the general form of the surfaces of the elastic limit (i = 1), maximum strength (i = 2), and residual strength (i = 3), we take the function $\tau_i(p)$ in the form (i = 1, 2, 3)

$$\tau_i = (Y_i + \alpha_i p)^{S_i} \quad (p \leqslant p_i^{(m)}),$$
$$\tau_i = \tau_i^{(m)} = \text{const} \quad (p > p_i^{(m)}),$$
$$\alpha_i = (Z_i - Y_i)/\tau_i^0, \quad Z_i = (\tau_i^0)^{1/S_i},$$
$$\tau_1^0 = \tau_y^0/\tau_f^0,\ \tau_2^0 = 1, \quad \tau_3^0 = \tau_r^0/\tau_f^0,$$

$$p_1^{(m)} \approx (2{,}3\alpha_1^{S_1})^{\zeta}, \quad \zeta = (1 - S_1)^{-1},$$
$$p_2^{(m)} = p_3^{(m)} \approx (\alpha_2^{S_2}/\alpha_3^{S_3})^{\nu}, \quad \nu = (S_3 - S_2)^{-1},$$

where τ and p are dimensionless quantities (τ_f^0 and $p_f^0 = (2/3)\tau_f^0$ are used as the parameters for conversion to dimensionless form); α_i and S_i are constants for each earth material. The numbers τ_y^0, τ_f^0, τ_r^0 are the values of the elastic limit and ultimate strength (maximum and residual) in uniaxial compression. The pressures $p_1^{(m)}$ and $p_2^{(m)} = p_3^{(m)}$ characterize the transition from Coulomb plastic states to Tresca–non-Mises states, respectively, during the initial stage of dilatational deformation and beyond the ultimate strength. In deriving the expression for $p_1^{(m)}$, we used the experimental result in [2] on the approximate independence of the specific value of the ratio $C^m = \sigma_1/\sigma_3 \approx 0.3$, characterizing this transition, on the type of earth material. It should be noted that the minimum value of the pressure $p = p_0$ at which flow of the material can still occur is determined on the hardening section from the condition $\tau_1(p_0) = \tau_2(p_0)$.

The hardening parameter is calculated from the formula

TABLE 2

Classification number	τ_f^0, MPa	τ_1^0	τ_3^0	A_1	$M^0 \cdot 10^{-3}$	Λ^0	Λ_*^0	a_0	$R_1 \cdot 10^3$
Sandstones									
P-0	120	0,74	—	20	—	1,44	—	0,63	61
D-9	143	0,70	—	19	—	0,71	—	0,93	51
P-4	146	0,34	—	—	—	—	—	—	—
P-03	143	0,73	—	21	—	1,20	—	1,60	49
P-01	126	0,85	—	19	—	1,27	—	1,29	59
P-02	159	0,85	—	13	—	1,07	—	0,23	51
P-5	202	0,74	—	6,4	—	—	—	—	—
P-026	74	0,78	—	20	—	1,10	—	0,64	30
P-021	46	0,72	—	40	—	1,22	—	0,80	47
VO	65	0,78	0,09	35	5,0	1,50	—	1,14	32
NVO	71	0,85	0,07	32	2,1	1,40	1,6	1,31	—
Nugget	150	—	—	—	—	1,85	—	2,30	40
D-8	66	0,77	—	26	—	1,58	—	0,76	51
P-04	39	0,62	—	33	—	1,90	—	1,20	32
D-12	79	0,71	—	—	—	0,94	—	0,54	—
Arcose	34	—	—	—	1,0	—	—	—	—
Granite									
Biotite	87	0,80	0,07	39	0,9	1,5	1,6	1,4	
Climax granodiorite	150	0,20	—	—	—	—	—	—	—
Westerly	110	—	—	—	—	1,42	—	0,93	—
Biotitic plagiogranite	145	0,85	0,02	—	1,4	—	—	—	—
Plagiogranite	167	0,86	0,02	—	2,4	—	—	—	—
Siltstones									
From holes 328	91	0,75	—	30	—	1,17	—	0,77	—
» 322	79	0,76	—	—	—	1,0	—	0,70	—
» 330	76	0,82	—	—	—	1,08	—	0,35	—
D-19	90	0,76	—	—	—	0,74	—	1,11	—
Marbles									
Marble II	38	0,80	0,05	25	0,3	1,2	1,3	1,15	—
Marble I	60	0,56	—	—	—	0,87	—	0,50	—
Karrarsk	38	—	0,13	—	0,3	—	—	—	—
Tenesse	16	—	0,16	—	18	—	—	—	—
Limestones									
D-6	92	0,85	—	20	—	0,64	—	0,30	—
Estonian shale	40	0,72	—	—	—	0,28	—	2,15	—
KMA	33	0,45	—	—	—	—	—	—	—
d'Fuville	16	—	0,32	—	9,0	—	—	—	—
Diabases									
№ 5	84	0,92	—	32	—	1,10	—	1,27	54
Bratsk hydroelectric power plant	150	0,88	0,01	39	4,5	0,92	—	1,00	59
Diorites									
№ 11	208	0,86	—	12	—	0,77	—	1,19	83
Quartz	125	0,64	—	24	—	0,81	—	0,44	32

$$\varkappa_1 = p_y^n \sqrt{(2\gamma^p - e^p)/3},$$

where p_y is the reference pressure, corresponding to point A of the intersection of the strain path with the elastic limit (see Fig. 1);

$$n = n_1 = \frac{S_1}{2(\tau_2/\tau_1 - 1)} \quad (p_y \leqslant p_1^{(m)}); \quad n = 0 \quad (p_y \geqslant p_2^{(m)});$$

$$n = n_1 \left(p_2^{(m)} - p_y\right) / \left(p_2^{(m)} - p_1^{(m)}\right) \quad \left(p_1^{(m)} < p_y < p_2^{(m)}\right).$$

TABLE 3

Coefficients	Sandstones								Siltstones		
	Y_1	S_1	S_2	τ_1^0	A_1	Λ^0	a_0	R_1	S_2	Λ^0	a_0
M_i	0,02	0,61	0,66	0,80	20	1,03	0,79	0,05	0,46	1,31	1,43
$N_i \cdot 10^3$	—1,1	6,3	5,6	—7,4	1100	50	51	—1,4	65	—140	—510
K, %	—	58	86	—	69	86	85	70	46	84	93

The softening parameter is determined in the form

$$\varkappa_2 = M p_f^{-q} \frac{\Delta(2\gamma^p - e^p)}{3}, \quad q = \frac{S_2}{2(1 - \tau_3/\tau_2)},$$

where p_f is the reference pressure corresponding to point B of the intersection of the strain path with the ultimate strength; $\Delta(2\gamma^p - e^p)$ is the increment of the quantity $(2\gamma^p - e^p)$ from its value at the ultimate strength. The decay modulus M is calculated from the formula

$$M = M^0 (p_2^{(m)} - p_f)^2/(p_2^m - 1)^2 \quad (p_f \leqslant p_2^{(m)}),$$
$$M = 0 \quad (p_f > p_2^{(m)}),$$

where M^0 is the decay modulus in uniaxial compression normalized on τ_f^0. The function $f_i(p)$ will be represented by the power expressions $f_i(p) = A_i p^{-m_i}$, $A_2 = -1$, $A_3 = 0$,

$$m_1 \approx n, m_2 \approx -q.$$

The invariant form of the dilatation rate on the hardening and softening sections is determined by the formula

$$\Lambda = \Lambda^0 \exp\{-a_0 \operatorname{sign}(R)\sqrt{|R|}\} \; (R \geqslant b \text{ or } R < b \text{ and } \tau \leqslant \tau_2).$$
$$\Lambda = \Lambda_*^0 \exp\{-a_1 \operatorname{sign}(R)\sqrt{|R|}\} \quad (R < b \text{ and } \tau \geqslant \tau_2).$$

Here, $R = (p - \tau)/3$ (in particular, if $\sigma_2 = \sigma_1$, then $R = -\sigma_1/\tau_f^0$); $a_1 = a_0 + \ln(\Lambda_*^0/\Lambda^0)/\sqrt{b}$; Λ^0 and Λ_*^0 are the dilatation rates in uniaxial compression to the left and right of the ultimate strength. At $R \geq b$, the rate does not undergo any sharp change when the ultimate strength is crossed. The parameter b is close to zero ($b > 0$), but there are still no experimental values of it.

The expression for the dilatation rate was obtained from analysis of the empirical data in [2, 3]. With a constant lateral pressure σ_1, the quantity Λ does not change during deformation up to the residual strength. At the residual strength, Λ abruptly decreases up to zero. At small values of $[\sigma_1]$, certain earth materials are characterized by a sudden increase in Λ with the crossing of the ultimate strength.

The elastic moduli K and G can be calculated as follows. There is almost no change in these moduli during deformation on the strain hardening section [4]. This is also evidently true of the residual-strength section. In a first approximation, the elastic moduli on the latter section can be found by linear interpolation of the residual moduli determined in [5] from tests in uniaxial compression and values of the moduli at $p \geq p_2^{(m)}$ (the latter coincide with the moduli of the unfractured material [6]). The elastic moduli decrease during deformation on the softening section and can be approximately determined by linear interpolation of the residual moduli and the moduli of the unfractured material (at the maximum strength).

3. To obtain the constants in the equations of the yield-point surface and the dilatation condition, we analyzed the results of static tests [2, 3, 7] conducted under conditions of constant lateral pressure or simple loading: Tables 1 and 2 (m_0 is porosity, $Y_1 \approx Y_2$, $Y_3 \approx 0$, $K_1 = K/\tau_f^0$, $G_1 = G/\tau_f^0$, $R_1 = R_p/\tau_f^0$, , R_p is technical cohesive strength, $R_c = 2\tau_f^0$). Tests of Hoggar granite [7] were conducted for very large values ($p < 72$). It turned out that no transition from Coulomb states to the Tresca state occurred even at these substantial pressures. For

TABLE 4

Parameters	Earth materials							
	1	2	3	4	5	6	7	8
m_0, %	0,25	0,6	1,2	0,8	1,6	5	9	2,6
τ_f^0, MPa	167	45	128	117	98	85	53	99
$K_1 \cdot 10^{-2}$	2,1	9	2,5	4,3	2,9	2,1	5,1	4,0
$G_1 \cdot 10^{-2}$	1,5	4,5	1,9	2,7	1,8	1,6	3,2	2,5
$Y_1 \cdot 10^3$	25	14	40	18	—	14	—	22
S_1	0,62	0,59	0,66	0,62	0,47	0,64	0,56	0,59
S_2	0,66	0,64	0,76	0,65	0,61	0,69	0,60	0,66
S_3	—	0,84	0,87	—	—	0,85	—	0,85
τ_1^0	0,75	0,68	0,84	0,87	0,77	0,73	0,65	0,75
τ_3^0	—	0,11	0,04	0,005	—	0,085	0,32	0,11
A_1	18	20	39	35	20	24	20	25
$M^0 \cdot 10^{-3}$	—	0,3	1,6	4,5	—	2,7	9,0	3,6
Λ^0	0,8	1,0	1,5	1,0	1,0	1,3	0,5	1,0
Λ_*^0	—	1,2	1,6	—	—	1,6	—	1,5
a_0	0,8	0,8	1,2	1,1	0,7	0,9	1,2	1,0
$R_1 \cdot 10^3$	58	—	50	56	—	46	—	53

Note: 1) diorites; 2) marbles; 3) granites; 4) diabases; 5) siltstones; 6) sandstones; 7) limestones; 8) values of the parameters averaged over all of the earth materials studied.

TABLE 5

Source	Earth materials								
	1	2	3	4	5	6	7	7'	8
[2]	66	64	76	65	61	69	60	—	66
[9]	73	59	78	73	65	71	59	65	68
[10]	—	—	79	—	—	74	67	66	72

Note: the numbers have the same meanings as in Table 4 (7' denotes claystones).

Hoggar granite, the parameter S_2 generally depends on p: $S_2 = 0.8$ ($p \leq 19$); $S_2 = [0.8(71 - p) + 0.65(p - 19)]/52$ ($19 < p \leq 71$).

The coefficient of variation of the strength properties of the investigated earth materials ranged from 50 to 75%, while the correlation coefficient was within the range 97-99.9%. There is an adequate amount of data for the sandstones and siltstones. Thus, the following correlation relations were obtained for these materials for the correlation between the parameters of the model and porosity: $W_i = M_i + N_i m_0$ ($i = 1, 2, \ldots, n$), where the values of the coefficients M_i and N_i and the corresponding correlation coefficients are shown in Table 3 (Nugget sandstone was excluded from the analysis). Table 4 shows model parameters averaged for each group of earth materials. This data can be used for approximate calculations if the type of material is not known.

Experiments performed in [8] make it possible to evaluate the effect of strain rate ε_3 on the parameters τ_f^0, τ_3^0, M^0, Λ^0, Λ_*^0. It turns out that the modulus M^0 decreases appreciably and strength increases somewhat with an increase in ε_3. The dilatation rate is slightly dependent on ε_3.

Some authors [9, 10] have suggested that the equations of the maximum-strength surface be written in the largest and smallest principal stresses. Here, the effect of the intermediate principal stress is completely ignored. The relations constructed in [9, 10] were reconstructed in an invariant form and the resulting values of the exponent $S_2 \cdot 10^2$ ($Y_2 \approx 0$) are given in Table 5. It is evident from the table that these results agree satisfactorily with each other and with the measurements in [2].

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NONAXISYMMETRIC SOLUTION BIFURCATION AND THE STABILITY OF SHELLS OF REVOLUTION WITH A SINGULAR PERTURBATION

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We are examining the phenomenon of branching of the modes of equilibrium of a thin shell without buckling. Engineers are often faced with the problem of choosing a shell structure from the desired stable state when it has several equilibrium positions. In engineering practice, such requirements are typical, for example, in the use of shells as protective exploding membranes, in pneumatic automation systems containing shell elements, etc. The difficulties encountered in analyzing this type of problem are directly related to one of the central problems in the membrane theory of shells — the existence of many stable modes for one value of the load parameter.

Here we analyze the post-critical deformation of nearly perfect thin elastic shells under the influence of pressure

$$\rho = q\eta + p,\ p^* = \min_n \{p_n\},$$

where q is a small scalar parameter; $\eta(\alpha, \beta)$ is a function characterizing the distribution of the perturbing pressure over the surface of the shell; $\{p_n\}$ are eigenvalues. The dependence of the branching of a nonaxisymmetric mode of loss of stability of a conical shell on the form of the η-function was established. It was found that nonaxisymmetric bifurcation is accompanied either by an explosion or by buckling. The phenomenon of buckling is characterized by the fact that attainment of the bifurcation point does not exhaust the load-carrying capacity of the shell. If it is energetically favorable, the nonaxisymmetric mode is seen in the static state [1].

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